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REDUCTION OF TURBULENT FRICTION UNDER LOCAL SURFACE HEATING

A. V. Kazakov, M. N. Kogan, and A. P. Kuryachii

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Introduction. The drag-reduction method based on heating of the surface of a flat plate near its leading edge was first suggested by Kazakov et al. in [1]. The total friction drag of the plate was reduced in this case by increasing the stability of gas flow heated near the leading edge and moving afterward over a colder surface. This resulted in a considerable elongation of the laminar flow region in the boundary layer. The laminar-turbulent-transition delay method proposed by Kazakov et al. in [1] was experimentally validated by Belov and Struminskii, et al. in [2, 3].

Nevertheless, for fairly large values of the characteristic Reynolds number, the turbulent part of the boundary layer is much more extended than the laminar part, and further reduction of the viscous drag is possible if one decreases turbulent friction. Thus, the search for methods of reducing the friction drag using various actions on the turbulent boundary layer is of great importance [4].

The turbulent friction on an isothermal surface with temperature T_w^* higher than the recovery temperature T_r^* is known to be smaller than on an adiabatic surface in flow [5, 6]. However, uniform heating of the entire surface involves considerable technical problems, in particular, the necessity of ensuring a reliable heat insulation for a large area to prevent heat losses inside the body. In addition, in this case the energy supplied for heating of the boundary layer exceeds the gain due to the friction-drag reduction.

In the present paper, the friction in a fully developed turbulent boundary layer on a flat plate is studied as a function of energy supply to one or several local regions of the surface, the remaining part of the surface being thermally insulated. In this case, as will be shown below, the integral friction coefficient is smaller by approximately a factor of 2 than in the case of heat energy supplied to the gas in the boundary layer and distributed uniformly over the entire surface.

Statement of the Problem. Let us consider an inviscid heat-conducting gas flow past a flat plate with velocity u_∞^*, density ρ_∞^*, and temperature T_∞^* at infinity. It is assumed that at a certain distance l^* from the plate leading edge the laminar boundary layer becomes turbulent, and the place where heat is supplied to the boundary layer is located in the developed-turbulent-flow region and has the length h^*. The parameters of an undisturbed turbulent boundary layer are completely determined by the Reynolds number $\mathrm{Re}_\theta = \rho_\infty^* u_\infty^* \theta_0^* / \mu_\infty^*$ (μ_∞^* is the dynamic viscosity in the incoming flow and θ_0^* is the momentum thickness immediately upstream of the heated region) and, as follows from calculations, are independent of the Reynolds number of the transition, beginning with $\mathrm{Re} = \rho_\infty^* u_\infty^* l^* / \mu_\infty^*$. For this reason, the choice of the Reynolds number is not essential, because it is responsible only for the position of the beginning of the heating region determined by a specified value Re_θ.

The temperature along the surface of the heating region and behind it markedly changes. Therefore, to describe the turbulent boundary layer, one should use turbulence models that allow a correct consideration of the hereditary effects in this boundary layer which are most clearly manifested in the high-gradient regions.

An adequate description of turbulent flows in these regions is difficult for several reasons. It is known that in the regions with high longitudinal gradients some points can appear where the Boussinesq hypothesis, which is used to close equations in numerous turbulence models, is violated.

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One should remember that most of the widely used two-parametric and more complicated turbulence models contain empirical constants and functions which were selected by comparing numerical results with experimental data obtained for an incompressible gas flow past a thermally insulated or isothermal surface [7, 8]. Later on, van Driest proposed to extend these models to compressible heat-conducting gas flows based on the assumption that the momentum and energy transfer processes are similar [9]. Only recently have turbulence models been developed which are suitable for an adequate description of the turbulent boundary layer in compressible gas with intense heat transfer over the immersed surface [10]. However, it is not conclusively proved that they can be efficiently used to describe the characteristics of a turbulent boundary layer on an essentially nonisothermal surface, in particular, with a stepwise variation in its temperature and with regions of high longitudinal temperature gradients and other functions of the flow, which was experimentally studied, for instance, by Carvin et al. in [6].

Nevertheless, there are simple algebraic turbulence models that, notwithstanding their locality, simulate fairly well parameters such as the friction coefficient and heat flux even in a nonequilibrium boundary layer with rather high gradients of the parameters along the surface. At the same time, their use requires much smaller computational resources. In particular, the two-layer algebraic model of Cebeci and Smith [11] yields a good agreement with experiment for boundary-layer calculations on an essentially nonisothermal surface [12]. This fact gives us hope that at least qualitatively reliable results can be obtained when this model is used to calculate the flows with heat supply, which are considered in the present paper.

For numerical calculations, it is more convenient to represent the turbulent boundary-layer equations in dimensionless form, which allows one to take into account considerable changes in the boundary-layer thickness caused by surface heating. For this purpose, the local displacement thickness of the boundary layer δ^* which is a function of the longitudinal coordinate x is chosen as a characteristic vertical coordinate. In this case, the dimensionless coordinate of the external boundary of the computation domain y_e remains constant under any action on the boundary layer, which facilitates substantially numerical calculations.

The dimensionless variables are introduced according to the relations

$$x = \frac{x^*}{l^*}, \quad y = \frac{y^*}{l^*\delta(x)}, \quad \delta = \frac{\delta^*}{l^*}, \quad u = \frac{u^*}{u^*_\infty},$$
$$V = \frac{v^*}{u^*_\infty \delta} - \frac{yu}{\delta}\frac{d\delta}{dx}, \quad \rho = \frac{\rho^*}{\rho^*_\infty}, \quad T = \frac{T^*}{T^*_\infty}, \quad \mu = \frac{\mu^*}{\mu^*_\infty}.$$

Here u^* and v^* are the longitudinal and transverse velocity components; T^* is the temperature, K; and the x^* coordinate is counted off from the leading edge of the plate.

In this case, the system of equations and boundary conditions for the turbulent compressible boundary layer has the form

$$\frac{\partial \rho u}{\partial x} + \frac{\rho u}{\delta}\frac{d\delta}{dx} + \frac{\partial \rho V}{\partial y} = 0, \quad \frac{1}{\mathrm{Re}\delta^2}\frac{\partial}{\partial y}\left[\left(\mu + \mu_t\right)\frac{\partial u}{\partial y}\right] = \rho u \frac{\partial u}{\partial x} + \rho V \frac{\partial u}{\partial x}, \quad \rho = \frac{1}{T},$$
$$\frac{1}{\mathrm{Re}\delta^2}\frac{\partial}{\partial y}\left[\left(\frac{\mu}{\sigma} + \frac{\mu_t}{\sigma_t}\right)\frac{\partial T}{\partial y}\right] = \rho u \frac{\partial T}{\partial x} + \rho V \frac{\partial T}{\partial y} - (æ - 1)\,\mathrm{M}^2_\infty \frac{\mu + \mu_t}{\mathrm{Re}\delta^2}\left(\frac{\partial u}{\partial y}\right)^2,$$
$$y = 0: \qquad u = 0, \quad V = 0, \quad \mu\frac{\partial T}{\partial y} + (æ - 1)\,\mathrm{M}^2_\infty \mathrm{Re}\sigma\delta q_w = 0 \quad (x_0 \leqslant x \leqslant x_0 + h), \tag{1}$$
$$\frac{\partial T}{\partial y} = 0 \qquad (x \geqslant x_0 + h),$$
$$y = y_e: \qquad u = 1, \quad T = 1, \quad \sigma = 0.72, \quad \sigma_t = 0.9, \quad \mu = T^{3/2}\frac{1 + 114/T^*_\infty}{T + 114/T^*_\infty}, \quad æ = 1.4,$$

where q_w is the heat flux referred to $\rho^*_\infty u^{*3}_\infty$, M_∞ is the Mach number in the incoming flow, x_0 is the coordinate of the beginning of the heating region, and $h = h^*/l^*$ is its nondimensional length.

According to the two-layer model of [11], the turbulent viscosity μ_t in the above variables is described by the formulas

$$\mu_{ti} = \delta \mathrm{Re} \rho \left| \frac{\partial u}{\partial y} \right| (ky)^2 \left[1 + \exp\left(-\frac{y^+}{A^+} \sqrt{\rho_w/\rho} \right) \right]^2 \gamma_t \gamma, \quad 0 \leqslant y \leqslant y_{\mathrm{layer}},$$

$$y^+ = y \frac{\rho}{\mu} \left[\delta \mathrm{Re} \left(\frac{\mu}{\rho} \frac{\partial u}{\partial y} \right) \right]^{1/2}, \quad A^+ = 26, \quad \mu_{t0} = \delta \mathrm{Re} \rho \alpha \left| \int_0^{y_e} (1-u)dy \right| \gamma_t \gamma, \quad y_{\mathrm{layer}} \leqslant y \leqslant y_e. \tag{2}$$

Here the subscripts i and 0 refer to the inner and outer boundary-layer regions, respectively, and subscript w to the immersed surface (wall).

Formula (2) includes the coefficients defined by the relations

$$\alpha = 0.0168 \frac{1.55}{1+\Pi}, \quad \Pi = 0.55 [1 - \exp(-0.243 Z_1^{1/2} - 0.298 Z_1)],$$

$$Z_1 = \mathrm{Re}_\theta / 425 - 1 \quad \text{for} \quad \mathrm{Re}_\theta > 425, \tag{3}$$

$$\gamma = [1 + 5.5 (y/y_0)^6]^{-1}, \quad \gamma_t = 1 - \exp\left[-\frac{3}{c^2} \mathrm{Re}^{0.66} (x-1)^2 \right], \quad c = 60 + 4.86 \mathrm{M}_\infty^{1.92}$$

(y_0 is the y coordinate of the point where $u = 0.995$).

The boundary between the inner and outer regions of the layer y_{layer} is determined from the condition of continuous turbulent viscosity.

The velocity and temperature profiles of an undisturbed boundary layer before the heating region are the initial conditions for studying the thermal effect on the boundary layer. To obtain these profiles, a self-similar flow in the laminar boundary layer was calculated. This flow is used as the initial conditions for $x = 1$. System (1)–(3) is then solved up to the cross section $x = x_0$ where the specified value Re_θ is reached.

The nonlinear problem (1)–(3) was solved numerically by the iteration method on the basis of the numerical approach of [13] with second-order accuracy along the y coordinate and with first-order accuracy along the x coordinate. The function $\delta(x)$, which enters the problem, is found in each cross section of the boundary layer using the iteration process

$$\delta^{(i)} = \delta^{(i-1)} y_d, \quad y_d = \int_0^{y_e} (1 - \rho u) dy \quad (i = 1, 2, \ldots), \quad \delta^{(0)}(x) = \delta(x - \Delta x),$$

where i is the iteration number and $\delta(x - \Delta x)$ is the thickness δ in the previous cross section.

Calculation Results. The turbulent boundary layer was calculated for $\mathrm{M}_\infty = 2$. The initial velocity and temperature profiles of the developed turbulent boundary layer in cross section x_0 corresponded to $\mathrm{Re}_\theta = 5000$. Calculations were performed for a completely thermally insulated surface and also in the presence of two (case 1) and ten (case 2) local regions of surface heating, and the total energy supplied to the flow and referred to $\rho_\infty^* u_\infty^{*3} l^*$ was $Q = 5 \cdot 10^{-4}$. For each heating region $\Delta x \leqslant h$, a constant heat flux $q_w = Q/(hN)$ was assigned, where N is the total number of heating regions of the plate surface which is subject to the condition $hN = 100\,\theta_0$.

Figure 1 shows the distributions of the relative magnitudes of the local friction coefficient C_f/C_{f0}, where C_{f0} is the local value of the turbulent friction coefficient for a thermally insulated surface without energy supply. In all figures, the ratio of the distance from the beginning of the first heating region $X = x - x_0$ to the momentum thickness in the cross section is plotted on the abscissa x_0; the dashed curve corresponds to two surface-heating regions (the length of each section is $h = 50\theta_0$) located one after the other, so that the beginning of the second region is at a distance of $250\,\theta_0$ from the beginning of the first region (case 1); the solid curve describes case 2 (energy supply in ten identical heating regions with length $h = 10\,\theta_0$), the beginnings of two neighboring heating regions being shifted with respect to each other at a distance of $50\,\theta_0$. The dot-and-dash curve in Figs. 1–4 shows the distribution of the corresponding parameter in the boundary layer that is not disturbed during heating.

As is seen from Fig. 1, at the beginning of each heating region the local friction coefficient increases, as compared with the friction on a completely thermally insulated surface, and only if the heating region

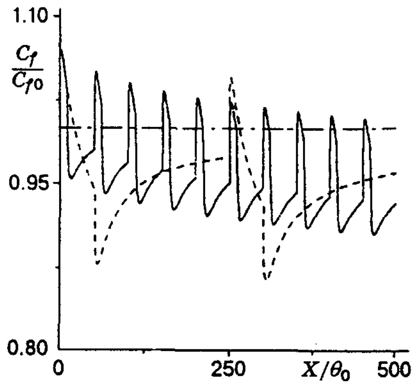

Fig. 1

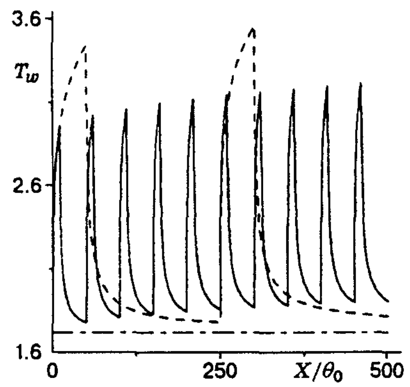

Fig. 2

is fairly long does the friction begin to decrease directly in the heating region itself, as in the case of two heating regions. On the thermally insulated surface which follows directly the region of energy supply to the flow, the friction further decreases. Farther downstream, the local friction coefficient on the thermal insulated surface begins to grow slowly, approaching monotonously the values that would occur in the corresponding cross sections of the boundary layer on a completely thermally insulated surface without energy supply.

This effect of energy supply on the friction coefficient, determined in the above dimensionless variables by the formula

$$C_f = \frac{2}{\delta \mathrm{Re}} \mu(T_w) \left(\frac{\partial u}{\partial y}\right)_w, \tag{4}$$

is mainly explained by the action of two competitive factors, namely, by the changes in viscosity and displacement thickness of the boundary layer under surface heating. Physically, the numerical results obtained are interpreted as follows. With an increase in the surface temperature, the dynamic viscosity also grows, which should give rise to an increase in C_f. On the other hand, the growth of the surface temperature T_w and heating of the near-wall gas jets reduce the gas density in the boundary layer, thus displacing the streamlines farther from the surface, reducing the gradient of the longitudinal velocity component $u_{yw} = (\partial u/\partial y)_w$, and increasing the boundary-layer displacement thickness $\delta(x)$.

The growth of the friction coefficient at the beginning of the heating region in all the cases considered is obviously explained by the prevailing effect of the first factor — the temperature dependence of the viscosity coefficient. Indeed, as is shown in Fig. 2, the temperature in the heating regions first increases quite rapidly, which leads to rapid heating of the near-wall flow jets and a higher molecular-viscosity coefficient $\mu(T_w)$. As the boundary layer is warmed up, the second factor starts to play a major part. This factor can become dominant even in the region of increasing surface temperature, as happens in the case of an extended heating region (case 1), indicated by the dashed curve in Fig. 2. A sharp reduction of the surface temperature behind the heating region leads to a considerable decrease in the dynamic viscosity. This gives rise to a considerable reduction of C_f in passing to the insulated section of the surface immediately adjacent to the heating region.

Figure 3 shows the effect of surface heating on the gradient of the longitudinal velocity component u_{yw}. Evidently, the velocity gradient in dimensionless variables decreases sharply in the heating region, reaching its minimum at the end. Since the dimensionless velocity gradient can be represented as $\partial u/\partial y = (\delta^*/u^*_\infty)\partial u^*/\partial y^*$, its reduction indicates that the dimensional velocity gradient decreases more rapidly than the boundary-layer displacement thickness grows. This is a natural consequence of the fact that, first of all, the flow jets in the immediate vicinity of the wall are heated and expanded, the near-wall velocity gradient decreases sharply, while the integral (total) displacement thickness increases more smoothly (Fig. 4). Since the wall is assumed to be thermally insulated downstream of the heating region, the overall heat supplied to the boundary layer remains in it, gradually heating the gas layers, which are more and more distant from the wall, along with the external-flow gas portions that enter the boundary layer. Thus, the displacement thickness

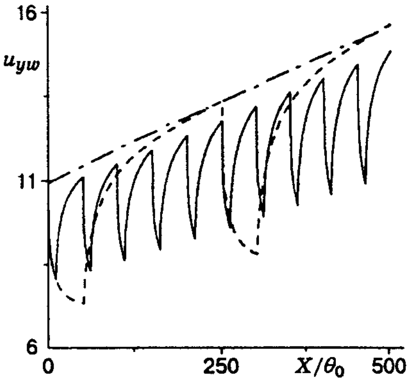

Fig. 3

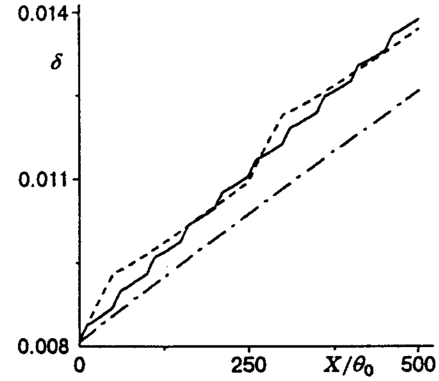

Fig. 4

slowly approaches asymptotically the value for a heated wall (Fig. 4), whereas the near-wall velocity gradient responds more rapidly to heat removal from the near-wall flow jets to the external part of the boundary layer (Fig. 3).

In other words, the behavior of the friction coefficient is explained by a competition between the viscosity variation as a function of the wall temperature and the variation of a certain effective (rather than the total) thickness of a more heated near-wall part of the boundary layer. Precisely, both the sharp increase in this effective displacement thickness in the heated region and its slower decrease in the thermally insulated part of the surface with a simultaneous marked reduction of the wall temperature lead to the fact that using the local heating, one can obtain a twofold reduction of drag, as compared with a uniformly heated surface (Fig. 5). Figure 5 shows the relative integral friction coefficient $\Delta C_F/C_{F0}$ determined by the relations

$$\Delta C_F = \int\limits_{x_0}^{x_0+X} (C_{f0} - C_f)dx, \quad C_{F0} = \int\limits_{x_0}^{x_0+X} C_{f0}dx.$$

From the viewpoint of a decrease in the total friction drag of the turbulent boundary layer, the behavior of the quantity $\Delta C_F/C_{F0}$ is of most interest at large distances downstream of the region where some extra energy is introduced into the boundary layer in this or that manner. The dot-and-dash curve in Fig. 5 refers to the relative integral friction coefficient $\Delta C_F/C_{F0}$ for a uniformly distributed supply of fixed energy Q along the plate surface from the initial cross section of the boundary layer x_0 to the current cross section X.

From Fig. 5 it follows that up to fairly large distances from the heat-supply region to the boundary layer, the integral decrease in friction for local heat supply is larger approximately by a factor of 2 compared with uniform surface heating. For uniform heat supply, the effective thickness of the boundary layer increases more slowly and is accompanied by a simultaneous growth of the near-wall gas viscosity, which seems to make the uniform heat supply less efficient. One can hope that a more rapid heat supply (namely, a smaller length of heating regions and an increase in their temperature) simultaneously with an increasing number of these regions will make the heat supply to the flow more efficient.

In fact, as is seen in Fig. 5, a larger number of heating regions located at a fixed part of the surface and their smaller length give rise to a slower reduction of $\Delta C_F/C_{F0}$ as one moves far away from the point of energy supply to the flow. The data presented testify to the fact that periodical local heating gives rise to the "accumulation" of thermal impacts on the boundary layer which are realized in each short heating region. This circumstance allows one to hope that C_f/C_{f0} values which are much less than unity can be reached on a thermally insulated surface in the relaxation wake behind the heating regions even at fairly large distances downstream if these regions are sufficiently large in number. This ensures a higher energy efficiency of the proposed thermal turbulent-friction reduction method.

It should be noted, however, that reducing the heating-region lengths and increasing the longitudinal temperature gradients and the other characteristics of the boundary-layer makes one less confident that both

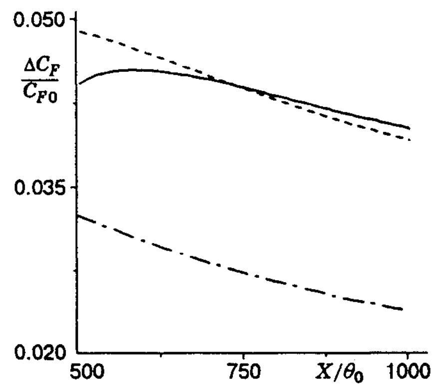

Fig. 5

the turbulence model used and, probably, the boundary-layer equations can be utilized to draw quantitative conclusions, at least in the boundary-layer regions close to very short heating regions. One can note the general trend to increasing the efficiency of friction-drag reduction under local heat supply. However, further optimization of heating and the final conclusions on the applicability of the proposed method for drag reduction should be postponed until the accuracy of the first results described above is experimentally verified and a turbulence model adequate for the problem is chosen.

In conclusion, we note that fairly high gradients of the boundary displacement thickness along the surface are observed in the heating region under local heat supply to the boundary layer, as is seen from Fig. 4. For this reason, the effect of the viscous–inviscid interaction on the friction-coefficient distribution can be profound. If the energy is supplied to the boundary layer on a fairly large number of heating regions, the boundary-layer displacement thickness has a wavy shape. The interaction of an external inviscid flow with such an effective wavy surface can stipulate an additional reduction of the friction drag [14]. Since in this case the actual surface experiencing the flow pressure remains flat, a possible decrease in the viscous drag is not accompanied by the occurrence of the pressure drag, as occurs in the case for the flow past actual wavy surfaces [14]. This effect is the subject of a separate study.

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